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Translated by M.D.F.

PMM U.S.S.R., Vol.48,No.2,pp.233-234,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

ON SPATIALLY HOMOGENEOUS RELAXATION IN THE DOMAIN OF HIGH MOLECULAR VELOCITIES*

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Asymptotic solutions of the Boltzmann equation are studied for the spatially homogeneous relaxation of the distribution function in the domain of fast molecules, as well as the evolution of the perturbations in the distribution function in the case of sherical scattering motion /1/. The problem was studied in the linear approximation in /2/, where exact solutions of the linearized Boltzmann equation were obtained for a specified form of molecular collision cross-sections.

Let us consider a spatially homogeneous gas composed of molecules, regarded as rigid spheres. We shall assume that at the initial instant we specify, on the range (ξ_{1*}, ξ_{2*}) of velocities,

$$\xi_{1*} \gg c = (2kT/m)^{1/2}, \ \Delta\xi = \xi_{2*} - \xi_{1*} \leqslant O(c^2/\xi_{1*})$$

a spatially homogeneous perturbation of the distribution function $\Phi(\xi)$ relative to the Maxwell distribution $f_0 = n(\pi c^2)^{-3/2} \exp(-\xi^2/c^2)$, i.e. $f = f_0 + \Phi$. We shall require that the following relation holds:

$$f_m = \max_{(\xi_{1*}, \xi_{2*})} \Phi(\xi) = O[f_0(\xi_{1*})] \tag{1}$$

and we shall have to explain how this perturbation evolves with time.

It was shown in /3, 4/ that the integral of elastic collisions $J(f, f)$ exhibits the following asymptotic behaviour at large velocities $\xi \gg c$:

$$J(f, f) = \int (f_1'f' - ff_{01}) g d\sigma d\xi_{01}, \quad g = |\xi_{01} - \xi_{02}| \tag{2}$$

since the fast molecules $(\xi \gg c)$ collide mainly with the "thermal" molecules moving with velocity of order c.

We find that the collision integral (2) can be simplified for the problem in question.

We shall denote the thermal molecules by X and the fast molecules by Γ. When the fast and thermal molecules (Γ, X) collide, we can have the molecules in the following states (Γ, X), (Γ, Γ), (X, Γ). Analyzing the dynamics of molecular collisions, with the molecules treated as rigid spheres, we can show that when $\xi \in (\xi_{1*}, \xi_{2*})$ we can neglect, within the range of collisions (2), the "glancing" $(\Gamma, X) \to (\Gamma, X)$ and "frontal" $(\Gamma, X) \to (X, \Gamma)$ collisions with an error of order $O(c^2/\xi^2)$, as $\xi \to \infty$.

Consider the product

$$f_1'f' = f_{01}f_0\left[1 + \frac{\delta f'}{f_0'} + \frac{\delta f_1'}{f_{01}'} + \frac{\delta f' \delta f_1'}{f_0' f_{01}'}\right]$$
$$(\delta f' = f' - f_0', \ \delta f_1' = f_1' - f_{01}')$$

Using the estimates $\delta f' \leqslant O(f_m)$, $\delta f_1' \leqslant O(f_m)$, and (1) and remembering that when $\xi \to \infty$ only collisions of type $(\Gamma, X) \to (\Gamma, \Gamma)$ remain, we reduce the collision integral (2) to the form (d is the diameter of the molecule)

*Prikl.Matem.Mekhan.,48,2,327-329,1984

$$J(f, f) = \frac{f_0 - f}{\tau}[1 + O(c^2/\xi^2)], \quad \tau^{-1} = \pi d^2 \int f_{01} g d\xi_1$$

Thus we must solve the following kinetic equation in the limit as $\xi \to \infty$:

$$\partial \delta f / \partial t = -\delta f/\tau, \ \delta f = f - f_0 \tag{3}$$

provided that $\delta f|_{t=0} = \Phi(\xi)$.

We assumed that the temperature and density of the gas appearing in the Maxwell distribution function f_0 were constant in the relaxation process. The solution of (3) has the following form in the velocity interval (ξ_{1*}, ξ_{2*}):

$$\delta f(\xi, t) = \Phi(\xi) \exp(-t/\tau) \tag{4}$$

The result obtained can be derived by analyzing the results obtained in /2/. The derivation given here and based on obtaining the asymptotic expression for the collision integral is much simpler.

Let us consider the molecules whose interaction potential varies as $U = U_0 (r_0/r)^{s-1}$. For such molecules the angle of deviation is defined by the dimensionless parameter $\rho = (b/r_0)[4(s-1) U_0/(mg^2)]^{1/(1-s)}$ where b is the aiming parameter. It can be shown that when $s = O(1)$, the process of relaxation in the problem in qeustion is determined by the weak collisions, when the change in the energy of the molecule during the collision $\Delta\varepsilon$ is either less than, or of the order of kT.

It is for this reason that the asymptotic expression for the collision integral cannot be obtained in the form (3), However, in the case of the potentials truncated in ρ ($\rho \in [0, \rho_{max}]$. $\rho_{max} = O(1)$) all molecular collisions are strong ($\Delta\varepsilon \sim m\xi^2$) and the contribution of the "frontal" collisions of the type $(\Gamma, X) \to (X, \Gamma)$ can be neglected with error $O(c^2/\xi^2)$. As a result, the cross-section depends on the relative velocity g according to the formula $\sigma = \sigma_0/g^{4/(s-1)}$ and the relaxation time is given, as $\xi \to \infty$, by $\tau^{-1} = n\sigma_0 \xi^{1-4/(s-1)}$. When τ is defined in such a manner, the asymptotic expression for the collision integral has the form (3).

It should be noted that, as we have already said, in the case of power potentials weak collisions prevail in the limit as $s = O(1)$, $\xi \to \infty$. When the relaxation of the perturbation f_0 of the type considered here is determined by strong collisions, the passage to molecules regarded as rigid spheres takes place in the limit as $\xi = \text{const} \gg c$, $s \to \infty$. Assuming that $U_0 = O(kT)$ and using the expression describing the deflection of a molecule by a small angle given in /5/, we can show that when the velocity of the molecule is fixed and the parameter s tends to infinity, the weak collisions are the decisive collisions for $\xi = o(cs^{-1/2}\bar{b}^{1-s})$ where $\bar{b} = b/r_0 > 1$, i.e. the corresponding range of velocities narrows rapidly as s increases. As a result, when $\xi = \text{const} \gg c$ and as s increases the role of strong collisions increases. The results obtained apply to the case of a mixture of gases. The asymptotic formula for the collision integral for fast particles obtained above can be used to study the relaxation of low intensity beams of fast particles in a real gas.

As was shown in /1/, we can construct a solution for the scattering motion by transforming the variables from the solution of Boltzmann's equation for the spatially homogeneous case. This assertion remains true also for the asymptotically exact solutions. In accordance with this transformation we must perform the following substitutions in (4):

$$t \to \frac{t_0}{3}\left(1 - \frac{t_0^3}{t^3}\right), \quad \xi \to \frac{t}{t_0}\left(\xi - \frac{x}{t}\right), \quad n \to n(t_0)$$

The solution exists for $t \geqslant t_0$.

Analogous results can also be obtained for molecules with truncated potential.

The author thanks V.S. Galkin, V.A. Zharov, M.N. Kogan and N.K. Makashev for discussing this paper.

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